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253. Proposed by PROFESSOR R. D. CARMICHAEL, Anniston, Ala.

Find the maximum number of real points of inflection in each of the quartic curves $y^2 = ax^4 \pm x^2 + \beta$, and find the necessary and sufficient relations between a and β for the existence of this number of points of inflection.

Solution by G. B. M. ZERR, A. M., Ph. D., 4243 Girard Avenue, Philadelphia, Pa.

$$y = \sqrt{[ax^4 \pm x^2 + \beta]}. \quad \text{Then } \frac{dy}{dx} = \frac{2ax^3 \pm x}{\sqrt{[ax^4 \pm x^2 + \beta]}}.$$

$$\frac{d^2y}{dx^2} = \frac{2a^2x^6 \pm 3ax^4(2a-1) + 6a^2\beta x^2 \pm \beta}{(ax^4 \pm x^2 + \beta)^{\frac{3}{2}}} = 0.$$

$$\therefore x^6 \pm \frac{3(2a-1)}{2a}x^4 + 3\beta x^2 \pm \frac{\beta}{2a^2} = 0.$$

$$\text{Let } x^2 = z, \quad \frac{3(2a-1)}{2a} = a, \quad 3\beta = b, \quad \text{and } \frac{\beta}{2a^2} = c. \quad \therefore z^3 \pm az^2 + bz \pm c = 0.$$

$$\text{Let } z = u \mp \frac{1}{3}a. \quad \text{Then } u^3 - (\frac{1}{3}a^2 - b)u \mp (\frac{ab}{3} - \frac{1}{27}a^3 - c) = 0, \quad \text{or } u^3 - Au \mp B = 0.$$

$\therefore$ When $A^3/B^2 < 27/4$, there are three real unequal roots.

$$\therefore \frac{[(2a-1)^2 - 4a^2\beta]^3}{[6a^2\beta(2a-1) - 2a\beta - (2a-1)^3]^2} < 1.$$

Then $y^2 = ax^4 + x^2 + \beta$ can have six real points of inflection, while $y^2 = ax^4 - x^2 + \beta$ can have but four. The values of a and β can reduce this number in both cases.

NUMBER THEORY AND DIOPHANTINE ANALYSIS.

122. Proposed by DR. L. E. DICKSON, The University of Chicago.

If p is prime, $(p^4-1)(p^2-1)$ has no factor of the form $1+p^3x$, $x > 0$, if $p > 2$; $(p^6-1)(p^4-1)(p^2-1)$ has no factor of the form $1+p^5x$, $x > 0$.

Solution by H. S. VANDIVER, Bala, Pa.

I. Assume that $(p^4-1)(p^2-1)$ is divisible by $1+p^3x$ where x is not zero, p any integer, not zero. By inspection we see that there exists a positive integer y , not zero, such that

$$(p^4-1)(p^2-1) = (1+p^3x)(1+p^2y).$$